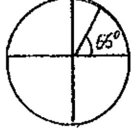
Pre-calculus 2nd Semester
Exam Review

Name ____________________

Chapter 5: The Trigonometric Functions

1) Find the angle that is coterminal with 775° and then find the reference angle.

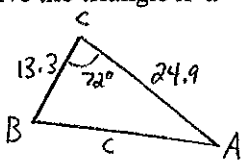


$775° - 360° = \boxed{415° \text{ - coterminal}}$ ← sample answer

or

$415° - 360° = \boxed{55° \text{ - reference}}$

2) Solve the triangle if $a = 13.3$, $b = 24.9$, and $C = 72°$.



Law of Cosines: $c^2 = 13.3^2 + 24.9^2 - 2(13.3)(24.9)\cos 72°$

$\boxed{c = 24.34}$

Law of Sines: $\dfrac{24.34}{\sin 72°} = \dfrac{24.9}{\sin B}$

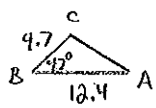
$\boxed{m\angle B = 76.68°}$

$\boxed{m\angle A = 13.32°}$

3) Find the area of the triangle with $a = 4.7$, $c = 12.4$, and $B = 47°$.

$\text{Area} = \frac{1}{2}(4.7)(12.4)\sin 47°$

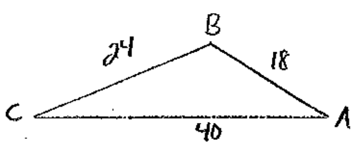
$= \boxed{21.31 \text{ units}^2}$



4) Solve the triangle when $a = 24$, $b = 40$ and $c = 18$.

Law of Cosines: $40^2 = 18^2 + 24^2 - 2(18)(24)\cos B$

$\boxed{m\angle B \approx 144.1°}$



Law of Sines: $\dfrac{40}{\sin 144.1°} = \dfrac{24}{\sin A}$

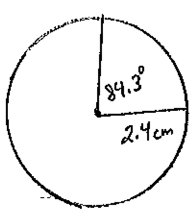
$\boxed{m\angle A = 20.6°}$

$\boxed{m\angle C = 15.3°}$

5) Change 42° to radian measure in terms of π.

$42°\left(\dfrac{\pi}{180°}\right) = \boxed{\dfrac{7\pi}{30}}$

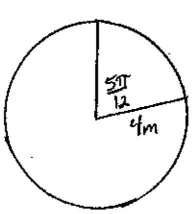
6) Given a central angle of 84.3 °, find the length of the intercepted arc in a circle of radius 2.4 cm.



$\dfrac{84.3°}{360°} = \dfrac{\ell}{2\pi(2.4)} \Rightarrow \boxed{\ell = 3.53 \text{ cm}}$

↑
Circum.

7) Find the area of the sector if the central angle measures $\dfrac{5\pi}{12}$ radians and the radius is 4 meters.



$\dfrac{\frac{5\pi}{12}}{2\pi} = \dfrac{A}{\pi(4)^2}$ $\boxed{A = 10.47 m^2}$

Chapter 6: Graphs of the Trigonometric Functions

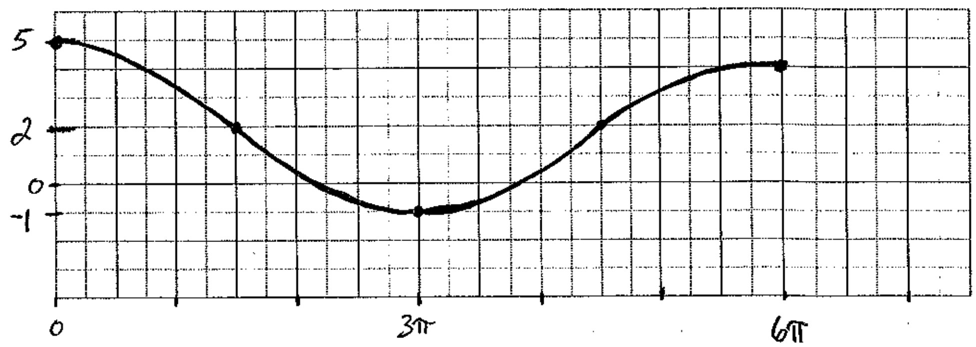
8) State the amplitude, period, phase shift and vertical shift of $y = 3\cos(\frac{6}{3}) + 2$. Then graph it.

amp = 3

period = $\frac{2\pi}{\frac{1}{3}} = 6\pi$

p.s. = 0

V.S. = 2



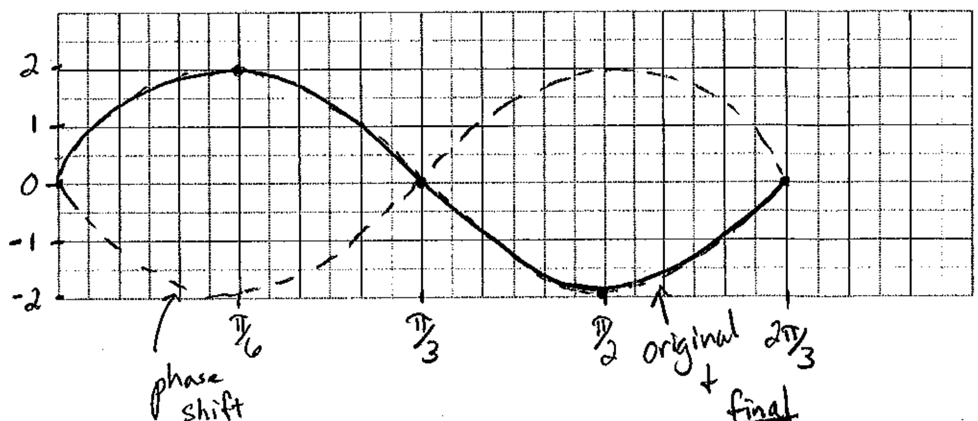
9) State the amplitude, period, phase shift and vertical shift of $y = -2\sin(36 + \pi)$. Then graph it.

amp = -2 (flip) → or 2

period = $\frac{2\pi}{3}$

p.s. = $-\frac{\pi}{3}$

V.S. = 0



10) Write an equation for the sine function with an amp 2.4, period 8.2, phase shift $\frac{\pi}{3}$, and a vertical shift 0.2.

$\boxed{a = 2.4}$

$p = 8.2 = \frac{2\pi}{k}$

$\boxed{k \approx .766}$

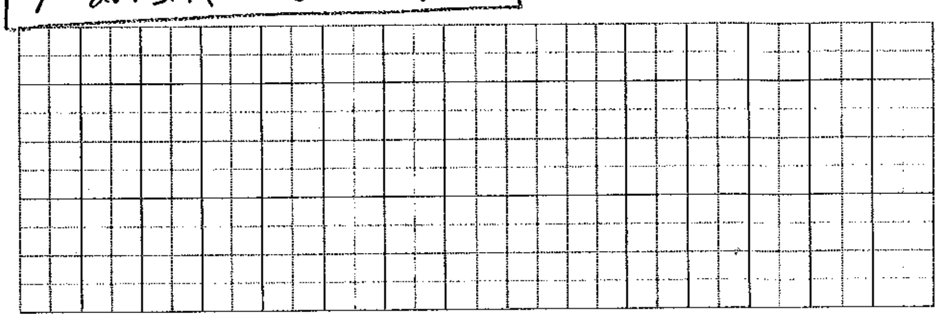
C: $\frac{\pi}{3} = \frac{-c}{k} = \frac{-c}{.766}$

$\boxed{c = -.802}$

$\boxed{V.S. = .2}$

$y = a\sin(k\theta + c) + v$

$\boxed{y = 2.4\sin(.766\theta - .802) + .2}$



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Chapter 7: Trigonometric Identities and Equations

11) No Calculator: Solve $2\sin^2 x - \cos x - 1 = 0$, over the set of real numbers.

$2\sin^2 x - \cos x - 1 = 0$
$2(1-\cos^2 x) - \cos x - 1 = 0$
$2 - 2\cos^2 x - \cos x - 1 = 0$
$-2\cos^2 x - 1\cos x + 1 = 0$

$2\cos^2 x + 1\cos x - 1 = 0$
$(2\cos x - 1)(\cos x + 1) = 0$
$2\cos x - 1 = 0$ $\quad \cos x + 1 = 0$
$\cos x = \frac{1}{2}$ $\quad \cos x = -1$

$\cos x = \frac{1}{2}$ $\quad \cos x = -1$
60, 300 $\quad$ 180

$\boxed{60° \pm 360° n \quad 180° \pm 360° n \quad 300° \pm 360° n}$

12) No Calculator: Simplify the expression and state the domain: $\sin x \sec x =$

$= \sin x \frac{1}{\cos x}$

$= \frac{\sin x}{\cos x}$ $\quad \cos x \neq 0$

$\boxed{= \tan x \qquad x \neq 90° \pm 180° n}$

13) No Calculator: Solve over the set of real numbers $(\sin x + 1)(\tan x - 1) = 0$

$\sin x + 1 = 0$ $\quad \tan x - 1 = 0$
$\sin x = -1$ $\quad \tan x = 1$
$x = 270°$ $\quad x = 45, 225$

225, 45

$\boxed{45° \pm 180° n \quad 270° \pm 360° n}$

14) No Calculator: Solve over the set of real numbers $\cos 2x + 7\sin x - 4 = 0$

$\cos 2x + 7\sin x - 4 = 0$
$1 - 2\sin^2 x$
$1 - 2\sin^2 x + 7\sin x - 4 = 0$
$-2\sin^2 x + 7\sin x - 3 = 0$
$2\sin^2 x - 7\sin x + 3 = 0$
$(2\sin x - 1)(\sin x - 3) = 0$

$2\sin x - 1 = 0$ $\quad \sin x - 3 = 0$
$\sin x = \frac{1}{2}$ $\quad \cancel{\sin x = 3}$
$x = 30, 150$

$\boxed{30° \pm 360° n \quad 150° \pm 360° n}$

15) No Calculator: Solve over the set of real numbers $\cos x = 0$

$\cos x = 0$
$x = 90, 270$

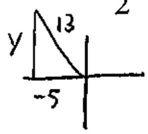
$\boxed{90° \pm 180° n}$

16) No Calculator: Solve over the set of real numbers $\sin 2x = ½$

$\sin 2x = \frac{1}{2}$
$2x = 30$ $\quad 2x = 150$
$x = 15$ $\quad x = 75$

$\boxed{15° \pm 360° n \quad 75° \pm 360° n}$

17) Given $\frac{\pi}{2} \le x \le \pi$ and $\cos x = -\frac{5}{13}$, find csc x.



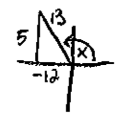
$\csc x = \frac{r}{y}$ or $\frac{1}{\sin x}$

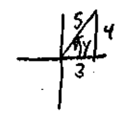
$(-5)^2 + y^2 = 13^2$

$y = 12$

$\boxed{\csc x = \frac{13}{12}}$

18) Suppose $\frac{\pi}{2} \le x \le \pi$ and $\sin x = \frac{5}{13}$. Also suppose $0 \le y \le \frac{\pi}{2}$ and $\cos y = \frac{3}{5}$.





a. Find sin $(x - y)$.

$\sin(x-y) = \sin x \cos y - \cos x \sin y$

$= \frac{5}{13} \cdot \frac{3}{5} - \frac{-12}{13} \cdot \frac{4}{5}$

$= \frac{15}{65} + \frac{48}{65}$

$= \boxed{\frac{63}{65}}$

b. Find cos $(x - y)$.

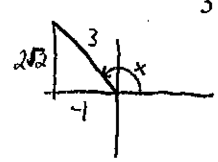
$\cos(x-y) = \cos x \cos y + \sin x \sin y$

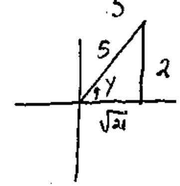
$= -\frac{12}{13} \cdot \frac{3}{5} + \frac{5}{13} \cdot \frac{4}{5}$

$= \frac{-36}{65} + \frac{20}{65}$

$= \boxed{\frac{-16}{65}}$

19) Suppose x and y are on the intervals $\frac{\pi}{2} < x < \pi$ and with $0 < y < \frac{\pi}{2}$, with $\cos x = -\frac{1}{3}$ and $\sin y = \frac{2}{5}$.





a. Find cos(x + y) $= \cos x \cos y - \sin x \sin y$

$= -\frac{1}{3} \cdot \frac{\sqrt{21}}{5} - \frac{2\sqrt{2}}{3} \cdot \frac{2}{5}$

$= \frac{-\sqrt{21}}{15} - \frac{4\sqrt{2}}{15} = \boxed{\frac{-\sqrt{21} - 4\sqrt{2}}{15}}$

b. Find sin (x – y) $= \sin x \cos y - \cos x \sin y$

$= \frac{2\sqrt{2}}{3} \cdot \frac{\sqrt{21}}{5} - \frac{-1}{3} \cdot \frac{2}{5}$

$= \frac{2\sqrt{42}}{15} + \frac{2}{15} = \boxed{\frac{2 + 2\sqrt{42}}{15}}$

c. Find sin (2x) $= 2 \sin x \cos x$

$= \frac{2}{1} \cdot \frac{2\sqrt{2}}{3} \cdot \frac{-1}{3} = \boxed{\frac{-4\sqrt{2}}{9}}$

d. Find cos (2y) $= 1 - 2\sin^2 y$

$= 1 - 2\left(\frac{2}{5}\right)^2 = 1 - \frac{8}{25} = \boxed{\frac{17}{25}}$

e. Find tan y $= \frac{\sin y}{\cos y} = \frac{\frac{2}{5}}{\frac{\sqrt{21}}{5}} = \frac{2}{\sqrt{21}}\left(\frac{\sqrt{21}}{\sqrt{21}}\right) = \boxed{\frac{2\sqrt{21}}{21}}$

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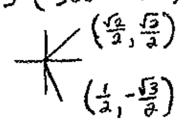
20) Find $\cos(345^\circ)$. $= \cos(300+45) = \cos 300 \cos 45 - \sin 300 \sin 45$



$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{-\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$

21) $\tan^{-1}(-\frac{\sqrt{3}}{3})$

$(-\frac{\sqrt{3}}{2}, \frac{1}{2}) + (\frac{\sqrt{3}}{2}, -\frac{1}{2})$ $\boxed{\theta = 150^\circ, 330^\circ}$

22) $\sin^{-1}(-\frac{\sqrt{3}}{2})$

$(\frac{1}{2}, \frac{-\sqrt{3}}{2}) + (\frac{-1}{2}, \frac{-\sqrt{3}}{2})$ $\boxed{\theta = 240^\circ, 300^\circ}$

23) $\cos^{-1}(-\frac{1}{2})$

$(-\frac{1}{2}, \frac{\sqrt{3}}{2}) + (-\frac{1}{2}, \frac{-\sqrt{3}}{2})$ $\boxed{\theta = 120^\circ, 240^\circ}$

24) No calculator: Give the exact value of $\cos\frac{5\pi}{12}$

$= \cos\left(\frac{3\pi}{12} + \frac{2\pi}{12}\right)$

$= \cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$

$= \cos\frac{\pi}{4}\cos\frac{\pi}{6} - \sin\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

or

$\cos 2x = 2\cos^2 x - 1$

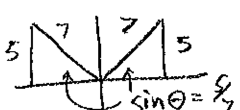
$\cos 2\left(\frac{5\pi}{12}\right) = 2\cos^2\frac{5\pi}{12} - 1$

$\cos\frac{5\pi}{6} = 2\cos^2\left(\frac{5\pi}{12}\right) - 1$

$-\frac{\sqrt{3}}{2} = 2\cos^2\frac{5\pi}{12} - 1 \Rightarrow \cos\frac{5\pi}{12} = \boxed{\frac{\sqrt{2-\sqrt{3}}}{2}}$

$\boxed{\frac{\sqrt{6}-\sqrt{2}}{4} = \frac{\sqrt{2-\sqrt{3}}}{2}}$

25) No calculator: Evaluate $\cos(\sin^{-1}\left(\frac{5}{7}\right))$

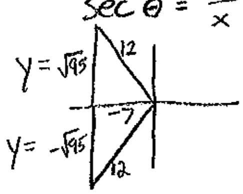


$\cos\theta = \frac{\pm\sqrt{24}}{7} = \boxed{\frac{\pm 2\sqrt{6}}{7}}$

$x = \sqrt{24}$ or $-\sqrt{24}$

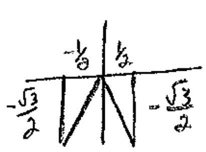
26) Give the exact value for $\sin(\sec^{-1}(-\frac{12}{7}))$

$\sec\theta = \frac{r}{x}$



$\sin\theta = \boxed{\frac{\pm\sqrt{95}}{12}}$

27) No Calculator: Evaluate $\tan(\sin^{-1}(-\frac{\sqrt{3}}{2}))$



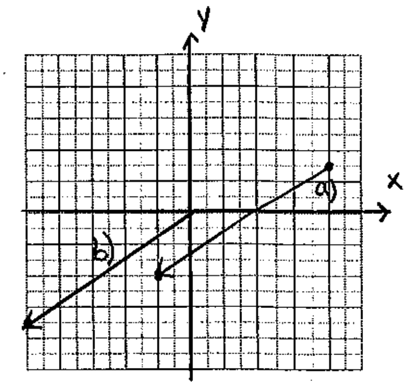
$\tan = \frac{y}{x}$ $\tan\theta = \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$ or $\tan\theta = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

$\boxed{\tan\theta = \pm\sqrt{3}}$

Chapter 8: Vectors and Parametric Equations

28) Using the vector, v, from (8, 3) to (-2, -4), answer the following questions:

a) Draw the vector v.



b) Draw the vector in standard position.

c) Write the components of v.

$\boxed{\langle -10, -7 \rangle}$

d) Write v as the sum of unit vectors.

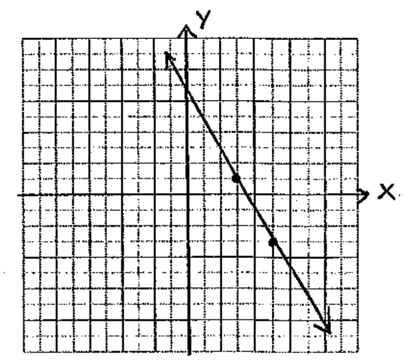
$\boxed{-10\hat{i} - 7\hat{j}}$

e) Write a vector that is orthogonal to v.

$\perp$ slope to $\frac{7}{10}$ is $\frac{-10}{7}$ so $\boxed{\langle 7, -10 \rangle \text{ or } \langle -7, 10 \rangle}$

29) Graph the line with parametric equations: $x = 5 - 2t$ and $y = -3 + 4t$.

$x = 5 - 2t$
$y = -3 + 4t$
point $(5, -3)$
vector $(-2, 4)$ slope $\frac{4}{-2}$



30) Write the parametric equations for the line through (1, -4) that is parallel to the line with vector equation $(x + 2, y - 3) = t\,(2, 3)$.

slope $m = \frac{3}{2}$

$\langle 2, 3 \rangle$ point $(1, -4)$

$$\boxed{\begin{aligned} x &= 1 + 2t \\ y &= -4 + 3t \end{aligned}}$$

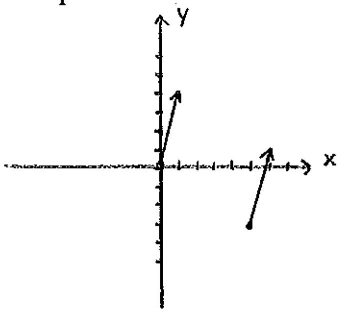
31) Write the parametric equations for the line through (1, -4) that is orthogonal to the line with vector equation $(x + 2, y - 3) = t\,(2, 3)$.

slope $\perp$ is $m = \frac{-2}{3}$

$\langle 3, -2 \rangle$ point $(1, -4)$

$$\boxed{\begin{aligned} x &= 1 + 3t \\ y &= -4 - 2t \end{aligned}}$$

32) Find the magnitude and direction of the vector that has initial point (5, -3) and terminal point (6, 1) **Hint**: Put in standard position first.



$m = \sqrt{1^2+4^2} = \sqrt{17}$

$\theta = \tan^{-1}\frac{4}{1}$

$\theta \approx 76^\circ$

$\boxed{m = \sqrt{17} \quad \theta \approx 76^\circ \text{ north of east}}$

33) Find the measure of the angle between **s** = (-1, 3) and **t** = (-2, -8).

$$\cos\theta = \frac{s \cdot t}{|s||t|} = \frac{2 + -24}{\sqrt{10}\cdot\sqrt{68}} = \frac{-22}{\sqrt{2^2\cdot 5\cdot 17}} = \frac{-22}{2\sqrt{170}} = \frac{-11}{\sqrt{170}}$$

$10 = 2 \cdot 5 \qquad 68 = 2 \cdot 34,\ 34 = 2 \cdot 17$

$$\cos^{-1}\left(\frac{-11}{\sqrt{170}}\right) \approx \boxed{147.53^\circ}$$

34) Given the equation $\vec{u}\times\vec{v} = (u_2v_3 - u_3v_2, u_3v_1 - u_1v_3, u_1v_2 - u_2v_1)$, find the cross product of vectors **y** = (3, -2, 7) and **z** = (1, -2, 1).

$(3, -2, 7) \times (1, -2, 1)$

$(-2 - -14,\ 7-3,\ -6 - -2)$

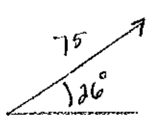
$\boxed{(12, 4, -4)}$

35) Given vector **k** = (-1, 2) and vector **v** = (10, 5), determine whether the two vectors are parallel, perpendicular, or neither.

$m_k = \frac{2}{-1} \qquad m_v = \frac{5}{10} = \frac{1}{2}$

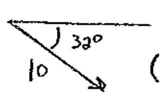
$\boxed{\text{perpendicular}}$

36) A boat travels at a speed of 75 mph at 26° north of east. It encounters a 10 mph current going 32° south of east. Estimate the resultant speed of the boat.



$(75\cos 26^\circ, 75\sin 26^\circ)$

$(67.4, 32.9)$



$(10\cos(-32^\circ), 10\sin(-32^\circ))$

$(8.5, -5.3)$

$(67.4 + 8.5,\ 32.9 - 5.3)$

$\boxed{(75.9, 27.6)}$

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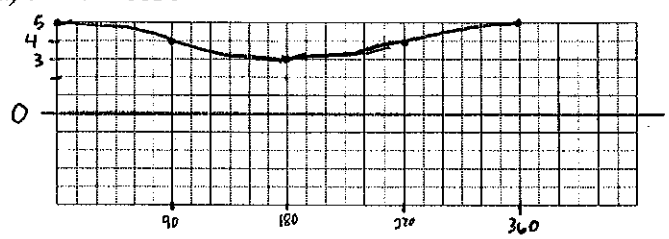
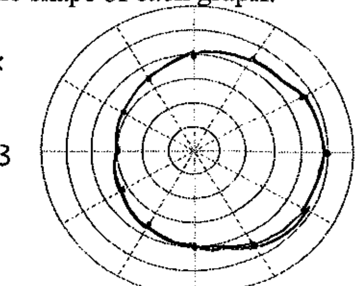
Chapter 9: Polar Coordinates and Complex Numbers

37) Sketch the rectangular and polar graphs of each equation below. Identify the shape of each graph:

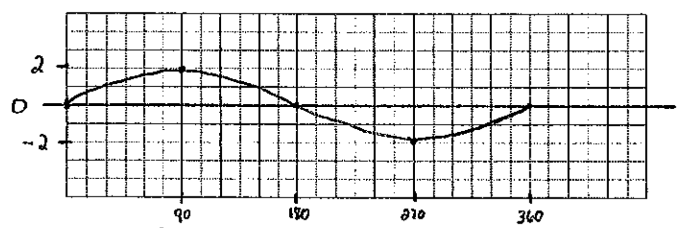
a) $r = 4 + \cos\theta$



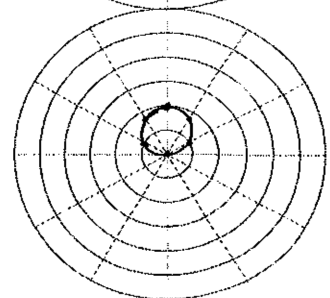
Symm. over x
tip: 4+1=5
dimple: 4-1=3
limaçon



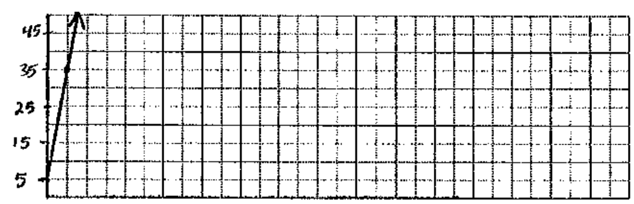
b) $r = 2\sin\theta$



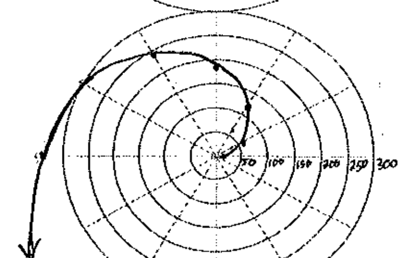
symm. over y
dia. 2
Circle



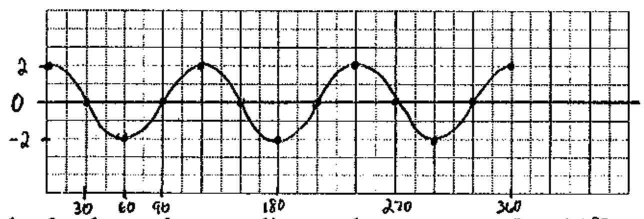
c) $r = 5 + 2\theta$



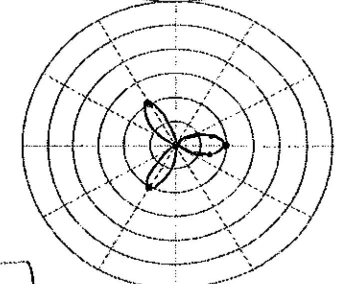
Spiral



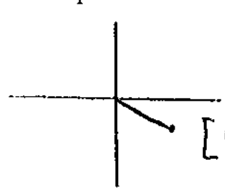
d) $r = 2cos(3\theta)$



length 2
# of petals = 3



38) Find the 3 other polar coordinates that represent [5, -30°].



[5, -30°]

$[5, 330°]$ $[-5, -210°]$
$[-5, 150°]$

39) Given the following polar coordinate, find it's equivalent coordinate in the other 3 forms (trigonometric, complex, and rectangular). [-5, 45°]. polar

$$\underbrace{-5(\cos 45° + i\sin 45°)}_{\text{trig}} = \underbrace{\left(-\frac{5\sqrt{2}}{2}, -\frac{5\sqrt{2}}{2}\right)}_{\text{rect.}} = \underbrace{-\frac{5\sqrt{2}}{2} - \frac{5\sqrt{2}}{2}i}_{\text{complex}}$$

40) Write (-2, -3) in the other 3 forms.

rect.

Quad III.

$$\underbrace{-2-3i}_{\text{complex}} = \underbrace{[\sqrt{13}, 236.3]}_{\text{polar}} = \underbrace{\sqrt{13}(\cos 236.3° + i\sin 236.3°)}_{\text{trig.}}$$

$\tan^{-1}\left(\frac{-3}{-2}\right) = 56.3$: put in Quad III.

41) If the rectangular coordinate of a point is $z = (9, -4)$, find z^4. $(9,-4)^4$

$$\left[\sqrt{9^2+(-4)^2},\ \tan^{-1}\left(\frac{-4}{9}\right)\right]^4$$

$$\left[\sqrt{97},\ -23.96°\right]^4 = \left[(\sqrt{97})^4,\ -23.96 \cdot 4\right] = \left[9409,\ -95.85\right]$$

$$= (9409 \cos -95.85°,\ 9409 \sin -95.85°)$$

$$= \boxed{(-957.4,\ -9360)}$$

42) Find all fifth roots of -3125.

$-3125 + 0i$ $\quad 360 \cdot \frac{1}{5} = 72°$ apart

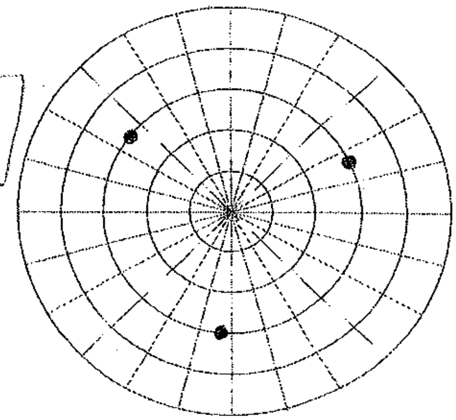
$$[3125,\ 180]^{\frac{1}{5}}$$

$$\boxed{= [5, 36°],\ [5, 108°],\ [5, 180°],\ [5, 252°],\ [5, 324°]}$$

43) Find the cube roots of 8 + 27i and graph them.

$$\left[\sqrt{8^2+27^2},\ \tan^{-1}\left(\frac{27}{8}\right)\right]^{1/3}$$

$$[28.16,\ 73.5°]^{1/3} = \boxed{[3.04,\ 24.5°],\ [3.04,\ 144.5°],\ [3.04,\ 264.5]}$$



Chapter 15A: Derivatives

44) The height (in feet) after t seconds of a batted ball with initial vertical velocity of 60 ft/sec is given by $h(t) = 60t - 16t^2$.

a. Find the average vertical velocity from time 2 seconds to time 3 seconds (include units).

Slope: $h(2) = 60(2) - 16(2)^2 = 56 \quad (2, 56)$

$h(3) = 60(3) - 16(3)^2 = 36 \quad (3, 36)$

$$m = \frac{56-36}{2-3} = \boxed{-20 \text{ ft/s}}$$

b. Find a formula for the slope of the **secant** line from time t to time $t + \Delta t$.

$$\frac{\Delta y}{\Delta x} = \frac{h(t+\Delta t) - h(t)}{\Delta t} = \frac{[60(t+\Delta t) - 16(t+\Delta t)^2] - [60t - 16t^2]}{\Delta t} = \frac{60t + 60\Delta t - 16t^2 - 32t\Delta t - 16\Delta t^2 - 60t + 16t^2}{\Delta t}$$

$$= \boxed{60 - 32t - 16\Delta t}$$

c. Find a formula for instantaneous velocity.

$$h'(t) = \boxed{60 - 32t}$$

d. What is the instantaneous velocity at $t = 2$? (Include units)

$$h'(2) = 60 - 32(2) = \boxed{-4 \text{ ft/sec}}$$

e. What is the instantaneous acceleration at $t = 2$? (Include units)

$$h''(t) = -32 \qquad h''(2) = \boxed{-32 \text{ ft/sec}^2}$$

45) Given $f(x)=\dfrac{-8x^2}{\sin x}$, find $f'(x)$.

Quotient Rule : $\dfrac{\text{Lo dHi} - \text{Hi dLo}}{D^2}$

Lo: $\sin x$ dLo: $\cos x$

Hi: $-8x^2$ dHi: $-16x$

$$f'(x)=\frac{(\sin x)(-16x)-(-8x^2)(\cos x)}{\sin^2 x}=\boxed{\frac{-16x\sin x+8x^2\cos x}{\sin^2 x}}$$

46) Use the limit definition of the derivative to calculate $f'(x)$ for $f(x)=4x+5$. Show all work.

$$\lim_{\Delta x\to 0}\frac{4(x+\Delta x)+5-(4x+5)}{\Delta x}=\lim_{\Delta x\to 0}\frac{4x+4\Delta x+5-4x-5}{\Delta x}=\lim_{\Delta x\to 0}\frac{4\Delta x}{\Delta x}$$

$$=\lim_{\Delta x\to 0}4=\boxed{4}$$

47) Write the equation of the tangent line for the function $f(x)=4x^3+5x^2-10$ at $x=2$. Show all work in determining this equation.

Point: $x=2$ $y=f(2)=4(2)^3+5(2)^2-10$

$y=42$

$(2,42)$

Slope: $f'(x)=12x^2+10x$ $f'(2)=12(2)^2+10(2)=68$

$(2,42)$ $m=68$

$y=mx+b$

$42=68\cdot 2+b$

$b=-94$

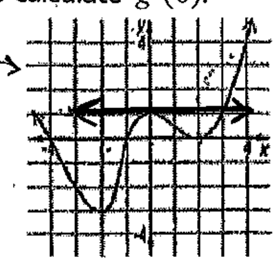
$\boxed{y=68x-94}$

48) Consider the function graphed at the right. Use the geometric definition of the derivative to calculate $g'(0)$.

Slope of tangent line at 0.

$g'(0)=0$

Slope = 0 →



49) Find a formula to calculate the slope of the line tangent to the function $f(x)=(6x+8)\cos x$.

Product Rule : 1d2 + 2d1

$$f'(x)=(6x+8)(-\sin x)+(\cos x)(6)$$

$$=\boxed{-6x\sin x-8\sin x+6\cos x}$$

50) Given $f(x)=\ln(12x+2x^2)$, find $f'(x)$.

Chain Rule: $u=12x+2x^2$

$u'=12+4x$

$f(u)=\ln u$ $f'(u)=\frac{1}{u}$

$$u'\cdot f'(u)=\frac{(12+4x)}{12x+2x^2}$$

Reduce $\boxed{\dfrac{6+2x}{6x+x^2}}$

41) If the rectangular coordinate of a point is $z = (9, -4)$, find z^4. $(9,-4)^4$

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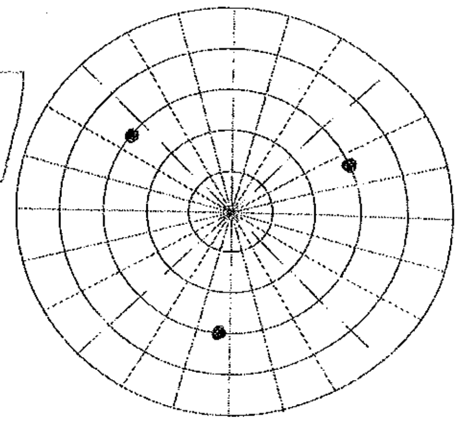

Chapter 15A: Derivatives

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53) Given $\int_0^5 (x^2+3)dx - \int_0^4 (x^2+3)dx$.

a) Use the properties of integrals to write the expression as a single integral.

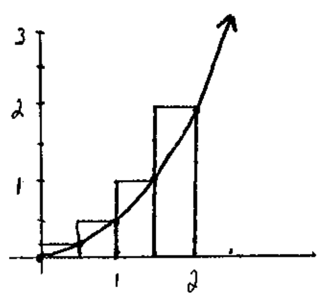
$$\boxed{\int_4^5 (x^2+3)dx}$$

b) Evaluate the integral.

$$\frac{x^3}{3} + 3x \Big|_4^5 = \left(\frac{5^3}{3} + 3(5)\right) - \left(\frac{4^3}{3} + 3(4)\right) = \boxed{23\tfrac{1}{3}}$$

54) Given the velocity function $f(x) = .5x^2$ on the interval $0 \le x \le 2$:

a) Sketch a picture of the situation.



b) Use Riemann Sums to estimate $\sum_{i=1}^{4} (f(z_i)\Delta x$ where z_i is the right endpoint of the ith subinterval

x	f(x)	Area
.5	.125	(.5)(.125)
1.0	.5	(.5)(.5)
1.5	1.125	(.5)(1.125)
2.0	2	(.5)(2)

width $\Delta x = .5$

height $f(x)$

Total = $\boxed{1.875}$

Pre-calculus 2^{nd} Semester
Exam Review

Chapter 15B: Integrals

51) No Calculator: Evaluate each

A) $\int_1^3 (\cos^2 x)dx + \int_1^3 (\sin^2 x)dx$

$\int_1^3 \cos^2 x + \sin^2 x \, dx$

$\int_1^3 1 \, dx = x\Big|_1^3 = 3-1 = \boxed{2}$

B) $\int_3^7 (x+3)dx + \int_1^3 (-x)dx$

$\frac{x^2}{2} + 3x \Big|_3^7 + \frac{-x^2}{2}\Big|_1^3$

$\left[\left(\frac{7^2}{2}+3(7)\right)-\left(\frac{3^2}{2}+3(3)\right)\right] + \left[-\frac{3^2}{2} - \frac{-1^2}{2}\right] = 32 + -4 = \boxed{28}$

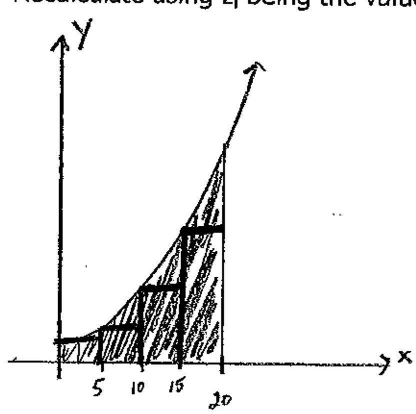
C) $\int_0^1 (\sqrt{1-x^2})dx + \int_0^6 (x^2)dx$

$\downarrow$

use picture $+ \frac{x^3}{3}\Big|_0^6$

$\frac{1}{4} \cdot \frac{\pi}{r^2} = \frac{\pi}{4} + \frac{6^3}{3} = \boxed{\frac{\pi}{4} + 72} \approx \boxed{72.79}$

52) For the function f(x) = $x^2 + 1$ Estimate the area bounded by the x-axis, y-axis, f(x) and x = 20 by partitioning the interval from 0 to 20 into 4 equal subintervals of equal length and using f(z_i) where z_i is the value of the right endpoint. Recalculate using z_i being the value of the left endpoint.



X	f(x)	Area
0	1	1.5
5	26	26.5
10	101	101.5
15	226	226.5

$\boxed{1770}$

left endpoint

X	f(x)	Area
5	26	26.5
10	101	101.5
15	226	226.5
20	401	401.5

$\boxed{3770}$

right endpoint

$\int_0^{20} x^2+1 \, dx$

$\frac{x^3}{3} + x\Big|_0^{20}$

$\frac{20^3}{3} + 20$

$\boxed{2686\frac{2}{3}}$

exact value